LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – NOVEMBER 2015

ST 1822 - STATISTICAL MATHEMATICS

Date : 07/11/2015 Dept. No. Max. : 100 Marks
Time : 01:00-04:00

Answer **all** the questions.

1. a) If $y_n \leq x_n \leq z_n$ for every n and $\lim_{n\to\infty} y_n = L = \lim_{n\to\infty} z_n$ then prove that $\lim_{n\to\infty} x_n = L$.

OR

b) Prove that $1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \cdots$ converges. **(5)**

c) (i) Let $\sum a_n$ be a divergent series of positive numbers. Then prove that there is a sequence $\{\varepsilon_n\}$ of positive numbers which converges to zero but $\sum \varepsilon_n a_n$ diverges.

(ii) Prove that a monotonic increasing sequence which is bounded above is convergent.

(10+5)

OR

d) (i) If $\{a_n\}$ is a decreasing sequence of positive terms converging to zero then prove that the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ converges.

(ii) Prove that the sequence $\left\{\frac{1}{n}\right\}$ has the limit $L = 0$. **(12+3)**

2) a) State and prove Taylor's formula with Lagrange's form of the remainder.

OR

b) If f is continuous at 'a' then prove that $|f|$ is continuous at a but the converse is not true.

(5)

c) If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ then prove that the following statements are true (i) $\lim_{x\to a}\big(f(x)g(x)\big) = LM$ (ii) $\lim_{x\to a}\left(\frac{1}{g(x)}\right) = \frac{1}{M}$ if $M \neq 0$. **(15)**

OR

d) (i) Prove that the number e is irrational.

(ii) State and prove inverse function theorem. **(7+8)**

3. a) If f' and g' are continuous on $[a,b]$, then prove that $\int_a^b f(x)\,g'(x)dx = f(b)g(b) - f(a)g(a) - \int_a^b f'(x)g(x)dx.$

OR

b) If f is continuous function on the closed bounded interval $[a,b]$ and if $\varphi'(x) = f(x)$ for $x \in [a,b]$ then prove that $\int_a^b f(x)dx = \varphi(b) - \varphi(a).$ **(5)**

c) (i) Let g be continuous on $[a,b]$ and f has a derivative which is continuous and never changes sign on $[a,b]$. Then prove that for some $c \in [a,b]$, $\int_a^b f(x)\,g(x)dx = f(a)\int_a^c g(x)dx + f(b)\int_c^b g(x)dx.$

(ii) If $f \in R[a,b]$ is continuous at $x_0 \in [a,b]$ and if $F(x) = \int_a^x f(t)dt$ where $a \le x \le b$ then prove that $F'(x_0) = f(x_0)$. **(10+5)**

OR

d) (i) Let f be bounded function on $[a,b]$. If P_1 and P_2 are any two partitions of $[a,b]$ then prove that $L[f;P_2] \le U[f;P_1]$.

(ii) If f is monotone on $[a,b]$ then prove that f is Riemann integrable on $[a,b]$.

(8+7)

4. a) If the rank of a matrix A of order (m,n) is r then prove that there is atleast one set of r linearly independent columns of A and every column can be written as a linear combination of any such set.

OR

b) State and prove Cauchy-Schwarz inequality. **(5)**

c) (i) Prove that the k n-vectors $A_1, A_2, \dots, A_k$ are linearly dependent if and only if the rank of the matrix $A = [A_1, A_2, \dots, A_k]$ with the given vectors as columns is less than k. Also prove that they are independent if and only if the rank is equal to k.

(ii) If the k n-vectors $A_1, A_2, \dots, A_k$ are linearly independent then prove that any $k+1$ linear combinations of these n-vectors are linearly dependent. **(8+7)**

OR

d) (i) Let V be a vector space over F, not consisting of the zero vector alone then prove that V contains atleast one set of linearly independent vectors $A_1, A_2, \ldots, A_k$ such that the collection of all linear combinations X of the form $X = t_1A_1 + t_2A_2 + \cdots + t_kA_k$ where $t's$ are arbitrary scalars from F, is precisely V. Moreover, prove that the integer k is uniquely determined for each V.

(ii) Find the complete solution of non-homogeneous system $x_1 - x_2 + 2x_3 = 1$ and $2x_1 + x_2 - x_3 = 2$. **(10+5)**

5 a) Prove that the characteristic roots of a real symmetric matrix are real.

OR

b) Apply the Gram Schmidt orthonormalization process to the vectors $(1,0,1)$, $(1,0,-1)$, $(0,3,4)$ to obtain an orthonormal basis for R^3. **(5)**

c) Reduce the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_1x_3$ to canonical form through an orthogonal transformation. **(15)**

OR

d) (i) Let $\lambda_1, \lambda_2, \ldots, \lambda_k$.be distinct characteristic roots of a matrix A and let $X_1, X_2, \ldots, X_k$ be any non zero characteristic vectors associated with these roots respectively then prove that $X_1, X_2, \ldots, X_k$ are linearly independent.

(ii) Find the inverse of the matrix $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$. **(8+7)**

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